ACTIVE RESONATOR AS A DYNAMIC ABSORBER OF ACOUSTIC VIBRATIONS IN A BOUNDED VOLUME

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One of the more effective devices used for the suppression of vibrations in engineering is the dynamic absorber [1, 2]. It has the advantage that the dimensions of the absorber can be small in comparison with those of the primary vibratory system. In particular, the Helmholtz resonator functions as a dynamic absorber when it is used for noise suppression. It should be noted, however, that the Helmholtz resonator in its classical form operates as a reactive absorber and has the drawback of a limited frequency range in which it works efficiently [3].

Here we discuss a planar model of a resonator, from which a gas jet issues and which is used to suppress acoustic vibrations in a bounded volume. Since part of the acoustic energy in this case is spent in the generation of vortices as the jet flows from the resonator, we say that such a resonator is active.

1. THE RESONATOR AS A DYNAMIC ABSORBER

The operating mechanism of the Helmholtz resonator as a dynamic absorber can be illustrated in a simple model. The Helmholtz resonator is modeled by a rectangular channel Ω_2, which is appended to one side of a rectangular region Ω_1 (Fig. 1), whose opposite side radiates acoustic energy at a given frequency ω. Introducing the assumptions

$$\varepsilon \ll l,\ \varepsilon \ll b \ll a, \tag{1.1}$$

we analyze the problem of determining the amplitude function φ of the velocity potential

$$\bar{\varphi}(t, x, y) = \varphi(x, y) \cos \omega t \tag{1.2}$$

in the region $\Omega = \Omega_1 \cup \Omega_2$, where the potential satisfies the Helmholtz equation

$$\Delta\varphi + k^2\varphi = 0 \tag{1.3}$$

($k = \omega/c_0$ is the wave number, and c_0 is the sound velocity), subject to the boundary conditions

$$\partial\varphi/\partial\nu = 0 \quad \text{for } (x, y) \in L' \tag{1.4}$$

(L' is the contour of the rigid boundary of the region, exclusive of the part x = a, and ν is the direction of the normal to L');

$$\partial\varphi/\partial x = 1 \text{ for } x = a,\ 0 < |y| < b/2; \tag{1.5}$$

$$\varphi = 0 \quad \text{for } x = -l,\ 0 < |y| < \varepsilon/2. \tag{1.6}$$

In the vicinity of resonance with the natural frequencies of the lowest vibrational mode in the region Ω_1, the approximation solution of problem (1.2)-(1.6) can be written in the form

$$\varphi_1 = A \cos kx \text{ in } \Omega_1; \tag{1.7}$$

$$\varphi_2 = B \sin k(x + l) \text{ in } \Omega_2. \tag{1.8}$$

Since Eqs. (1.7) and (1.8) are approximate, the relation between the functions φ_1 and φ_2 is not established by matching them at the common part of the boundary of the regions Ω_1 and Ω_2, but is determined from the energy conservation law [4]

$$\frac{\partial}{\partial t}\iint\limits_{\Omega_j} E\, d\sigma + \int\limits_{L_j} \mathbf{I}\cdot\boldsymbol{\nu}\, ds = 0. \tag{1.9}$$

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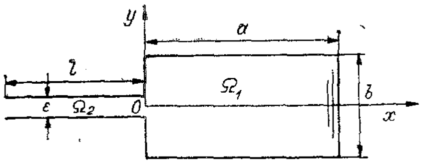

Fig. 1

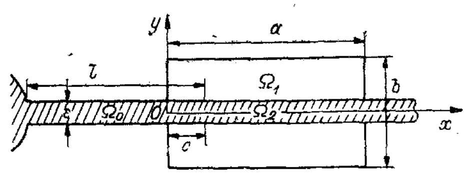

Fig. 2

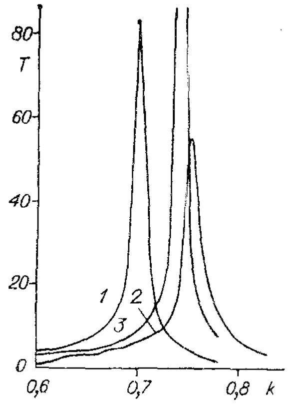

Fig. 3

Here E is the acoustic energy density, **I** is the intensity vector (acoustic energy flux), and **ν** is the outward normal to the contour L_j of the region Ω_j ($j = 1, 2$).

We write the energy flux from the resonator into Ω_1 approximately as

$$J = -\rho_0 \int\limits_{-\varepsilon/2}^{\varepsilon/2} \frac{\partial \varphi_1}{\partial t} \frac{\partial \varphi_2}{\partial x} dy = -\frac{1}{2} \rho_0 \omega k \varepsilon \cos kl \cdot \sin 2\omega t \cdot AB. \tag{1.10}$$

Taking Eq. (1.10) into account and applying Eq. (1.9) to Ω_1, we obtain

$$bk \sin 2ka \cdot A - 2k\varepsilon \cos kl \cdot B = -2b \cos ka. \tag{1.11}$$

Applying Eq. (1.9) to Ω_2, we have

$$2\varepsilon \cos kl \cdot A - b \sin 2kl \cdot B = 0. \tag{1.12}$$

Solving the system (1.11), (1.12), we obtain

$$A = -2b^2 \cos ka \cdot \sin 2kl/D; \tag{1.13}$$

$$B = -4b^2 \cos ka \cdot \cos kl/D, \tag{1.14}$$

where

$$D = k(b^2 \sin 2ka \cdot \sin 2kl - 4\varepsilon^2 \cos^2 kl). \tag{1.15}$$

We note that this solution corresponds exactly with the theory of dynamic absorption [1, 2]. Above all, when the resonator frequency $\omega_r = \pi c_0/\ell$ coincides with the natural frequency in the main region $\omega_0 = \pi c_0/a$ at resonance, so that $\omega = \omega_0 = \omega_r$, the amplitude $A \to 0$, i.e., the vibrations are completely suppressed. The suppression mechanism follows from the expression (1.10) for the acoustic energy flux from the resonator, which is equal in absolute value and opposite in sign to the acoustic energy flux from the external source when the value of $B = b/\varepsilon k$ is taken into account in this regime. Moreover, the solution (1.13), (1.14) reflects the shortcoming of dynamic absorbers operating without energy dissipation. The problem is that such absorbers are effective only in a narrow frequency range. Thus, by setting D (1.15) equal to zero in the case of "well-tuned" resonators, such that $\bar{\delta} = \omega_r/\omega_0 - 1 \ll 1$, where the resonator volume is sufficiently small in comparison with the volume of the main region Ω_1, i.e., $\bar{\varepsilon} = \varepsilon/b \ll 1$, we find that $A \to \infty$ at $\omega \approx \omega_0 \left[1 + \frac{1}{2}\left(-\bar{\delta} \pm \sqrt{\bar{\delta}^2 + 4\bar{\varepsilon}^2/\bar{k}_0^2}\right)\right]$ ($\bar{k}_0 = \omega_0 b/c_0$ is the normalized natural frequency in Ω_1).

2. ACTIVE RESONATOR MODEL

The efficiency of the resonator when it is used to absorb acoustic energy in a certain volume can be increased substantially by causing a gas jet to issue from the throat of the resonator. In this case the part of the acoustic energy arriving from the external source is spent in vortex generation.

To estimate qualitatively the influence of a jet on the resonator efficiency, we consider a planar model of acoustic vibrations in a certain rectangular region Ω_1 and an active resonator Ω_0 (with a gas jet issuing from it) attached to this region (Fig. 2). We assume for simplicity that the gas jet issues from the resonator at a velocity U without expanding and escapes from the main region through a hole at the opposite end of the region Ω_1. We consider the resonator to be situated in an arbitrary position, described by the parameter c, on the symmetry axis of this region.

Let a source of acoustic vibrations with frequency ω be located on the right boundary of Ω_1. We investigate the problem of determining the amplitude function of the transient component of the pressure in the domain $\Omega = \Omega_0 \cup \Omega_1$

$$p'(t, x, y) = p(x, y)\, e^{j\omega t}, \tag{2.1}$$

which satisfies the equation

$$(1 - \overline{M}^2)\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - 2jk\overline{M}\frac{\partial^2 p}{\partial x^2} + k^2 p = 0, \tag{2.2}$$

where

$$M = \begin{cases} 0 & \text{at} \quad \varepsilon/2 < |y| < b/2, \\ M & \text{at} \quad |y| < \varepsilon/2, \end{cases}$$

$k = \omega b/2c_0$ is the reduced frequency of the vibrations, $M = U/c_0$ is the Mach number, and x, y and all the geometrical parameters (a, b, ε) are assumed everywhere to be dimensionless, referred to b/2, with the following boundary conditions:

the condition on the acoustic energy flux from the external source

$$\frac{\partial p}{\partial x} = \mu \,\mathrm{th}\, \mu a \cdot \sin\frac{\pi}{2}|y| \quad \text{at} \quad x = a,\ |y| > \varepsilon/2 \ \left(\mu = \sqrt{(\pi/2)^2 - k^2}\right); \tag{2.3}$$

the impenetrability condition

$$\partial v/\partial \nu = 0,\ (x, y) \in L/L_1 \tag{2.4}$$

(v is the amplitude function of the transient component of the gas velocity, L is the contour of the rigid boundary of the entire region, and L_1 is the right boundary of the region Ω_1);

the approximate condition on the open parts of the boundary of Ω

$$p = 0 \quad \text{at} \quad x = c - l,\ a;\ |y| < \varepsilon/2; \tag{2.5}$$

the Zhukovskii–Kutta condition at the edges of the resonator

$$[p] = 0 \ \text{at points} \ x = c,\ y = \pm\varepsilon/2; \tag{2.6}$$

the condition on the lines of contact discontinuity ($c < x < a$, $|y| = \varepsilon/2$) [4]

$$[p] = 0,\ ikv_{1y} + M\partial v_{1y}/\partial x = ikv_{2y} \tag{2.7}$$

(v_1 is the amplitude of the transient component of the gas velocity at $y > \varepsilon/2$, and v_2 is the same at $y < \varepsilon/2$).

The function v in the boundary conditions (2.4) and (2.7) can be expressed in terms of the function p by means of the Cauchy–Lagrange integral, which in our case is reducible to the form

$$p = -\rho_0 c_0 (ik\varphi + \overline{M}\partial\varphi/\partial x) \tag{2.8}$$

(φ is the velocity potential amplitude function).

3. METHOD OF SOLUTION

By geometrical symmetry, we assume that the unknown function p is also symmetric about the x axis, i.e.,

$$\partial p/\partial y = 0 \quad \text{at} \quad y = 0. \tag{3.1}$$

Adopting Eq. (3.1) as the boundary condition, we formulate the unknown function for the upper half of the region Ω. To do so, we partition it into three subregions: Ω_1, which is bounded by the straight lines ($x = 0$, a; $y = \varepsilon/2$, 1); Ω_2, which is bounded by the lines ($x = c$, a; $y = 0$, $\varepsilon/2$); Ω_0, which is bounded by the lines ($x = c - \ell$, c; $y = 0$, $\varepsilon/2$). In each of these subregions we represent the unknown function by series whose terms satisfy Eq. (2.2) and the corresponding boundary conditions (2.3)-(2.7), (3.1), viz.:

in $\bar{\Omega}_0$

$$p_0 = e^{ik\beta Mx}\left[c_0\left(e^{-ik\beta x} - e^{2ik\beta(l-c)}e^{ik\beta x}\right) + \sum_{m=1}^{\infty} c_m e^{\lambda_m(x-c)}\cos\frac{2\pi m y}{\varepsilon}\right] \tag{3.2}$$
$$\left(\beta = \frac{1}{1-M^2},\ \lambda_m = \frac{1}{\varepsilon}\sqrt{(2\pi m)^2 - (k\varepsilon)^2}\right);$$

in $\bar{\Omega}_1$

$$p_1 = \frac{\operatorname{ch}\mu x}{\operatorname{ch}\mu l}\sin\frac{\pi}{2}y + \sum_{m=0}^{\infty} a_m \frac{\operatorname{ch}\bar{\lambda}_m(1-y)}{\operatorname{ch}\bar{\lambda}_m(1-\varepsilon/2)}\cos\bar{m}x \tag{3.3}$$
$$\left(\bar{m} = \pi m/a,\ \bar{\lambda}_m = \sqrt{\bar{m}^2 - k^2}\right);$$

in $\bar{\Omega}_2$

$$p_2 = e^{ik\beta Mx}\left[c_0 g\left(e^{-ik\beta x} - e^{-2ik\beta a}e^{ik\beta x}\right) + \sum_{m=1}^{\infty} e^{-\lambda_m(x-c)}\cos\frac{2\pi m}{\varepsilon}y + \sum_{n=1}^{\infty} b_n \frac{\operatorname{ch}\tilde{\lambda}_n y}{\operatorname{ch}\tilde{\lambda}_n\varepsilon/2}\sin\tilde{n}(x-c)\right] \tag{3.4}$$
$$\left(\tilde{n} = \frac{\pi n}{a-c},\ \tilde{\lambda}_n = \sqrt{\tilde{n}^2 - k^2},\ g = \frac{1-e^{2ik\beta l}}{1-e^{-2ik\beta(a-c)}}\right).$$

The set of functions p_0, p_1, and p_2 represents the solution of Eq. (2.2) subject to conditions (2.3)-(2.7), (3.1), except on the part of the boundary $(0 < x < c,\ y = \varepsilon/2)$ of the region $\bar{\Omega}_1$. The arbitrary constants in Eqs. (3.2)-(3.4) are determined from the condition for matching of the functions p_0 and p_2:

$$\partial p_0/\partial x = \partial p_2/\partial x \quad \text{at} \quad x = c,\ 0 < y < \varepsilon/2; \tag{3.5}$$

from the first condition (2.7)

$$p_1 = p_2 \ \text{at}\ c < x < a,\ y = \varepsilon/2; \tag{3.6}$$

from the second condition (2.7) in conjunction with (2.4) on the section $(0 < x < c,\ y = \varepsilon/2)$

$$v_{1y} = \begin{cases} 0 & \text{at}\ \ 0 < x \leqslant c, \\ \dfrac{ik}{M} e^{-i\frac{k}{M}x}\displaystyle\int_c^x e^{i\frac{k}{M}x} v_{2y}\,dx + v_{1y}\left(c, \frac{\varepsilon}{2}\right) & \text{at}\ \ c < x < a \end{cases} \tag{3.7}$$

(v_{1y} and v_{2y} are the projections of the amplitudes of the transient components of the velocities in the regions $\bar{\Omega}_1$ and $\bar{\Omega}_2$ onto the y axis).

From the Cauchy–Lagrange integral (2.8) we find

$$v_{1y} = \frac{i}{\rho_0 c_0 k}\frac{\partial p_1}{\partial y}; \tag{3.8}$$

$$v_{2y} = -\frac{e^{-i\frac{k}{M}x}}{\rho_0 c_0 M}\int_c^x e^{i\frac{k}{M}x}\frac{\partial p_2}{\partial y}dx + v_{2y}\left(c, \frac{\varepsilon}{2}\right). \tag{3.9}$$

It follows from condition (2.6) that

$$v_{1y} = v_{2y} = 0 \quad \text{at}\ x = c,\ y = \varepsilon/2. \tag{3.10}$$

Consequently, making use of Eqs. (3.8) and (3.9) and taking Eq. (3.10) into account, we transform Eq. (3.7) as follows:

$$\frac{\partial p_1}{\partial y} = \begin{cases} 0 \quad \text{at}\ \ 0 < x < c,\ \ y = \dfrac{\varepsilon}{2}, \\ -\dfrac{k^2}{M^2} e^{-i\frac{k}{M}x}\displaystyle\int_c^x\int_c^x e^{i\frac{k}{M}x}\frac{\partial p_2}{\partial y}dx \quad \text{at}\ \ c < x < a,\ \ y = \dfrac{\varepsilon}{2}. \end{cases} \tag{3.11}$$

Forming the Fourier series expansion of the functional relations (3.5), (3.6), and (3.11) on the corresponding intervals, we obtain an infinite system of algebraic equations in the unknown constants a_m, b_m, and c_m. It can be shown that the coefficients of this system satisfy conditions such that the system can be solved by reduction.

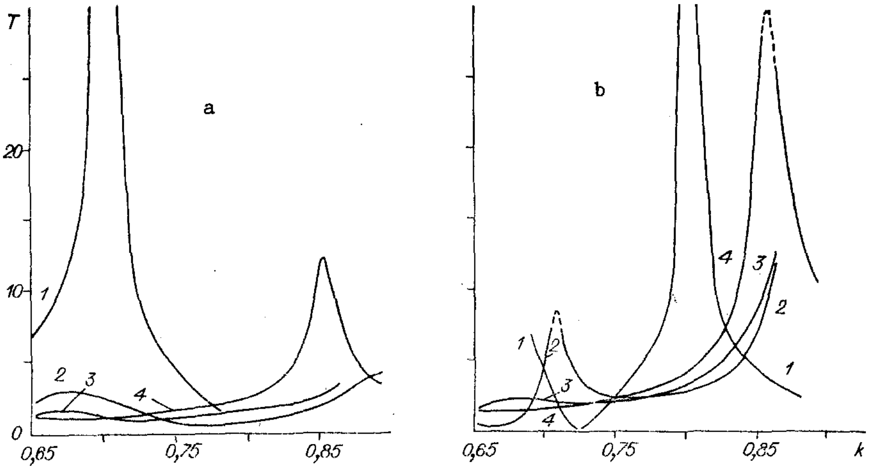

Fig. 4

4. SOME RESULTS OF CALCULATIONS AND THEIR ANALYSIS

The acoustic pressure function p in the region Ω can be analyzed as a function of the normalized frequency parameter k of the external source for various values of M and the geometrical parameters of the region ε, a, c, and ℓ implementing numerically the foregoing schematically described solution of problem (2.2)-(2.7). We adopt the following as such a function, which well characterizes the acoustic vibration level in the region Ω_1:

$$T = \int_{\varepsilon/2}^{1} [p(0, y) - p(a, y)]\, dy.$$

Figure 3 shows the numerical dependence of T on k for M = 0.05, where the resonator essentially operates as a reactive unit, for c = 0.05, ε = 0.2, a = 5, and ℓ = 3.5, 5, and 6 (curves 1-3, respectively). We see that when the parameter k is equal to its resonance value, $k_0 \approx \pi/5$, the function T(k) has a minimum in the region Ω_1 and increases sharply in the vicinity of k_0. This fact is consistent with the results of the approximate resonator theory set forth in Sec. 1.

The influence of the Mach number on the resonator efficiency is illustrated in Fig. 4, where T(k) is plotted numerically for c = 0.05, ε = 0.2, a = 5, M = 0.05, 0.2, 0.3, and 0.4 (curves 1-4), and ℓ = 3.5 (a) and 4 (b).

It is evident from Fig. 4 that the acoustic vibration level in the main region drops considerably in the presence of a jet, indicating that part of the acoustic energy is dissipated. This justifies treating the investigated resonator as an active absorber of acoustic energy. Part of the acoustic energy is evidently lost because of its conversion into transient vortex streets shed from the edges of the resonator. The generation of these streets in the given model is accounted for by conditions (2.6) and (2.7). This effect is also confirmed indirectly by the fact that the function T(M), all other conditions begin equal, is not monotonic. Indeed, since the intensity of the vortex streets generally increases with M, the given interpretation dictates that the dissipation of acoustic energy must also increase.

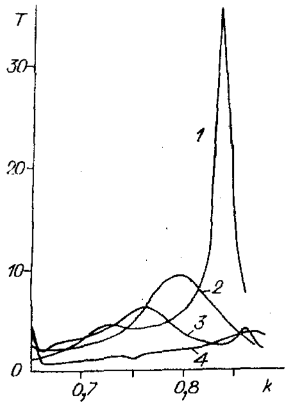

Fig. 5

Also, we know from the theory of the dynamic absorber [2] that its efficiency depends nonmonotonically on the damping.

Figure 5 shows the function T(k) for M = 0.2, ε = 0.2, a = 5, ℓ = 4.5, and c = 0.05, 2, 3, and 4 (curves 1-4); this figure exhibits the major influence of the position of the resonator in the region Ω on the absorption of acoustic vibrations. The mechanism of this influence can be identified with the displacement of the field of acoustic disturbances radiated from the resonator relative to the normal mode of acoustic vibrations in the main region.

In conclusion the author is grateful to V. A. Yudin for writing the program for the calculations.

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INITIATION OF COHERENT MOTION IN TURBULENT COCURRENT FLOWS

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It has been established experimentally that organized motion is present in all turbulent shear flows. The presence of coherent motion in flows served as the basis for Townsend's creation [1, 2] of a turbulence model with a binary structure in which the undisturbed surrounding fluid is brought into the shear flow by coarse eddies which develop against a background of small-scale turbulence. Townsend also developed the hypothesis of the universal similitude of free shear flows. In accordance with this hypothesis, at a sufficiently great distance from the source, motion is determined by the local scales of velocity and length. The scales depend on the type of flow and the external velocity and length scales. The average motion, referred to the local scales, is described by universal functions which depend only on the method by which the motion comes about. Coarse eddies are in dynamic equilibrium with the average flow. This subsidiary condition determines the form and intensity of these eddies. Similitude has been proven to exist for plane shear layers [3-6], plane wakes [7-9], axisymmetric wakes [10-14], and axisymmetric shear layers and plane jets [15]. Here, characteristic local values of velocity and length are used as the scales. However, such scales depend to a significant extent on the experimental conditions (the presence of small harmonic perturbations [4-6, 8] and external turbulence [16] and, for cocurrent flows, the form of the body [7-14]) and other features of the experiment. The type of load and its characteristic frequency and scale are reflected in the coherent structures present in these flows. Some authors [6, 8] have attempted to describe external effects by using the theory of hydrodynamic stability of inviscid flows. This theory can be used to analyze the response of a small harmonic perturbation.

The memory of the initial conditions by the flow is a generally recognized factor as well, at least for the ranges which have been studied. However, it is not yet clear whether or not universal asymptotic similitude exists for each type of free shear flow. It is difficult to explain the absence of such similitude in turbulent flows as being the result of intensive energy transfer between different scales of motion. Coherent large-scale structures have been recorded in developed turbulent flows at very large distances from the source. The mechanism of their reproduction may be hydrodynamic instability of the average flow. If a turbulent shear flow is modeled as a flow with a certain effective viscosity ν_t, then the corresponding turbulent Reynolds numbers (Re_t) will be finite and will determine whether the flow will be stable or unstable against longwave perturbations. At values of Re_t less than the critical value, the flow will be stable, and degeneration of small-scale turbulence will result in a decrease in ν_t and a consequent increase in Re_t. The flow will

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